DIQ-MPM: Dual Interface Quadrature MPM for Simulating Large Deformation and Fluid-Solid Coupling - Supplementary

1 Deformation Gradient Update and Hessian of TLMPM

Deformation gradient $\mathbf{F} = \frac{\partial \psi}{\partial \mathbf{X}} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}$ is used to characterize finite deformation in continuum. In TLMPM, we update particle's $\mathbf{F}_p$ using $\mathbf{F}_p^{n+1} = \mathbf{F}_p^n + \Delta t \frac{\partial \mathbf{F}^n}{\partial t}$, where $\frac{\partial \mathbf{F}_p^n}{\partial t} = \frac{\partial}{\partial t}(\frac{\partial \mathbf{x}_p^n}{\partial \mathbf{X}}) = \frac{\partial \mathbf{v}_p^n}{\partial \mathbf{X}}$. In the total Lagrangian formulation, particle's velocity gradient $\frac{\partial \mathbf{v}_p^n}{\partial \mathbf{X}}$, also denoted as $\nabla^{\mathbf{X}} \mathbf{V}_p^n$, could be directly obtained on the initial grid:

$$\frac{\partial \mathbf{v}_p^n}{\partial \mathbf{X}} = \sum_i \mathbf{v}_i^n (\nabla^{\mathbf{X}} W_i(\mathbf{X}_p))^T. \tag{1}$$

We compute gradients directly on the initial grid, without relying on the current configuration, making the update scheme distinct from the conventional EMPM.

The internal force of TLMPM is $\mathbf{f}_i = -\sum_p V_p^0 \mathbf{P}(\mathbf{F}_p) \nabla^{\mathbf{X}} W_i(\mathbf{X}_p)$. The action of the energy Hessian on an arbitrary increment $\delta \mathbf{U}$ can be expressed as

$$-\delta \mathbf{f}_i = \sum_p V_p^0 \mathbf{A}_p \nabla^{\mathbf{X}} W_i(\mathbf{X}_p), \tag{2}$$

where

$$\mathbf{A}_p = \delta \mathbf{P}(\mathbf{F}_p) = \frac{\partial^2 \Psi}{\partial \mathbf{F} \partial \mathbf{F}} : \sum_j \delta \mathbf{U}_j (\nabla^{\mathbf{X}} W_i(\mathbf{X}_p))^T. \tag{3}$$

As can be seen, the form of $\mathbf{A}_p$ in TLMPM closely resembles that of EMPM [1], where $\mathbf{A}_p = \frac{\partial^2 \Psi}{\partial \mathbf{F} \partial \mathbf{F}} : \sum_j \delta \mathbf{u}_j (\nabla w_i(\mathbf{x}_p))^T$. The same similarity holds for $\delta \mathbf{f}_i$. Therefore, implicit TLMPM can be implemented by directly modifying the particle-grid transfer components of existing implicit EMPM frameworks for hyperelastic materials.

References

[1] A. Stomakhin, C. Schroeder, L. Chai, J. Teran, and A. Selle, "A material point method for snow simulation," *ACM Transactions on Graphics (TOG)*, vol. 32, no. 4, pp. 1–10, 2013.

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