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Visual Simulation of Multiple Fluids in Computer Graphics: A State-of-the-Art Report

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Abstract Realistic animation of various interactions between multiple fluids, possibly undergoing phase change, is a challenging task in computer graphics. The visual scope of multi-phase multi-fluid phenomena covers complex tangled surface structures and rich color variations, which can greatly enhance visual effect in graphics applications. Describing such phenomena requires more complex models to handle challenges involving calculation of interactions, dynamics and spatial distribution of multiple phases, which are often involved and hard to obtain real-time performance. Recently, a diverse set of algorithms have been introduced to implement the complex multi-fluid phenomena based on the governing physical laws and novel discretization methods to accelerate the overall computation while ensuring numerical stability. By sorting through the target phenomena of recent research in the broad subject

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Keywords physical simulation, multiple fluid, computer graphics

1 Introduction

Among many diverse phenomena in the physical world, the motion of varying fluids is a common, familiar element that brings visual vitality to a virtual environment. Fluid simulation techniques have made significant progress due to both rapid increase of computing power and recent development of more efficient algorithms. In computer graphics, fluid simulation plays as one of the key visual effects in animation and special effects, and also finds its application in virtual reality.

Earlier research in fluid simulation focused on a wide variety of single-fluid phenomena and numerical techniques, supporting the recreation of gaseous smoke, fire, water, and highly viscous liquid etc. For such tasks that contain only one type of fluid in the simulation, numerous algorithms have been dedicated to achieving visually plausible simulations, enhancing numerical stability and visual fidelity, while introducing simulation details to the visual appearance at lower computational costs. Thanks to these pioneering works, e.g. [1, 2, 3, 4, 5, 6], impressive visual effects can be efficiently reproduced using single-fluid simulation frameworks for many phenomena. Comprehensive description for single-fluid simulation techniques can be found in books e.g. [7, 8]. However, as real-world phenomena often involve more than one fluid or a single phase, existing single-fluid simulation techniques are generally insufficient to faithfully reproduce more complex multi-fluid interaction, such as two-way coupled gas-liquid interaction, interfacial flow formed by immiscible water and oil, fluid mixture as solution or suspension, bubble dynamics, chemical reactions, and phase transition etc. As a result, multiple-fluid or multi-phase simulation techniques have received increasingly more attention within the past decade, and the current multi-fluid literature covers a wide range of related phenomena using different methodologies. By sorting through the target phenomena of recent research in the broad subject of multiple fluid, this state-of-the-art report summarizes recent advances on multi-fluid simulation in computer graphics.

Conceptually, the terms “multiple fluid”, and “multiple phase” often appear in comput-

er graphics literature, but are not always used with consistent meanings. We first give a formal definition of the related concepts. In a narrower scope, the term "phase" denotes distinct physical states (such as gas, liquid or solid). However, it is usually unnecessary to make a distinction between phases and components/materials (such as water, oil, etc.) in multiple fluid models. For simplification, we use the term "phase" for a unified meaning as any different kinds of components/materials or distinct physical states in the survey except in specific names such as "phase transition". The concept of "multiple fluid" simulations in computer graphics also needs further specification, as it can easily be confused with "liquid" simulations. Liquids naturally involve a second gas phase, but are often treated as single-phase approximations for efficiency and stability reasons, e.g. [9]. They are often treated with a so-called "free surface" model, in which the gas dynamics are assumed to be negligible for the motion of the liquid. Certain works concentrate on solid-liquid coupling on the boundary where solid is only static or follows rigid-body dynamics. We exclude these research from our "multiple fluid" concept similarly because only one liquid phase is governed by models of fluid dynamics.

In this survey, we categorize a simulation framework as "multiple fluid" (or "multi-fluid" in short) if it satisfies all of the following conditions: 1) The phenomenon of interest consists of two or more types of physically distinct phases; 2) dynamic influence of more than one phases is calculated in the model; 3) interaction mechanism between phases is explicitly modeled with complete sets of equations. We further define the concept of "multiple phases" (or "multi-phase") as consisting of two or more phases and its dynamics calculated in the simulation. Using the above unified concept for "phase", we can see "multi-fluid" is always "multi-phase", but "multi-phase" is not necessarily "multi-fluid". Typical examples of "single-fluid multi-phase" include works on phase transition, such as [10, 11]. In these works, the mechanisms of different phases are described by changing attributes (density, viscosity, stiffness) under different states in the same equations, and they usually concentrate on describing how the attributes change. At the same time, no explicit interaction between the attribute-defined phases are considered. A diagram demonstrating the relations between the above concept is shown in Fig.1.

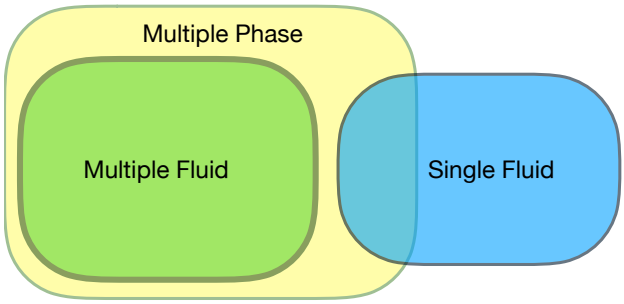

Fig. 1. Demonstration of the multi-fluid concept. Multi-fluid methods are always multi-phase, but not vice versa. Most of the single-fluid multi-phase examples fall into the phase transition topic.

The multiple fluid literature in computer graphics is shaped by target phenomena. On the one hand, while recovering the vastly different real-world multi-fluid phenomena, a large diversity of methods have been applied and many of them only appear in their specific papers, having relatively little connections between them. On the other hand, different categories of multi-fluid phenomena have different emphasis in their graphics modeling. As a result, this report groups the current works based on their target phenomena. We first recap the theoretical fundamentals of multi-fluid simulation in §2 to provide general categorizations of multi-fluid phenomena. Next, different categories of commonly used numerical methods for multi-fluid simulation implementation are summarized in §3. Then we introduce how each category of multi-fluid phenomena given in §2 are studied in computer graphics in the remaining chapters.

2 Fundamentals of Multi-Fluid Phenomena

In this section, we first examine phenomena involving multiple fluid materials, then we describe general physically-based models and techniques for reproducing multi-fluid effects in computer graphics.

Distinct phases of multiple fluids coexisting in the simulation generally show two kinds of behavior: "immiscible" behavior, i.e. phases tend to be apart from each other unmixed; or "miscible" behavior, i.e. the involved phases freely mix with each other. In many cases, different phases exhibit either purely immiscible or purely miscible behavior in the bulk volume, such as in water-oil or water-dye systems. The former forms vigorously evolving interfaces between different immiscible phases, and the latter usually features smoothly changing material properties and continuously varying color details. Consequently, the main concern in simulating bulk immiscible behavior lies in precise interface tracking [12, 13, 14, 15], while the main concern in simulating bulk miscible behavior lies in properly describing and capturing the state and the evolution of fluid mixture [16, 17, 18].

However, more complex cases exist, where

two originally immiscible phases can get temporarily mixed due to fluid motion (after stirring, splashing, or violent shaking, etc.) or artificial settling and forming dispersed state of one phase in another, such as a large amount of small bubbles in the water. Tracking each interface of every single dispersed part can become prohibitively costly. One approach is to "average" the dispersed parts on a macroscopic level and represent them using miscible descriptions [19, 20].

Another dimension of complexity comes from state changes of one or more phases during multiple-fluid interactions, such as in erosion-formed suspension, dissolving or reactive fluids [21, 22, 23]. Some phase transition works derive their theory in a multi-fluid environment [24, 25], and in several boiling and vaporization works, newly generated gas is treated as a distinct gas phase in the simulations [12, 26, 27].

We classify most of literature in computer graphics on multi-fluid simulations into each of the four categories: bulk non-mixing models in §4, bulk mixing models in §5, bubble dynamic models in §6, and state-change related multi-fluid models in §7. Given the extensive research in computer graphics on bubble formation, an independent section §6 is devoted to reflect its importance. Single-fluid multiphase phenomena are covered in §8. An additional summary on reconstructing methods for multi-fluid phenomena using capturing or non-physical approaches is also presented in §9.

Various physical models and techniques have been studied to reproduce the multiple fluid phenomena in graphics applications. We start with the single-fluid Navier-Stokes equation [28] written as

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla p + \mathbf{g} + \frac{1}{\rho}\nabla \cdot \mathbf{T} \qquad (1)$$

where $\mathbf{u}, \rho, p, \mathbf{g}$ are fluid velocity, density, pressure and gravity (and other accelerations), respectively. The viscous stress tensor is denoted as $\mathbf{T}$. Mass conservation for a single fluid is written as

$$\frac{D\rho}{Dt} + \rho\nabla \cdot \mathbf{u} = 0. \qquad (2)$$

$\frac{D}{Dt}$ denotes the material derivative which describes the time rate of change of a quantity at a fluid element that is advected with the flow. In Lagrangian formulations, e.g. Smoothed Particle Hydrodynamics (SPH), the material derivative simply becomes $\frac{D}{Dt} = \frac{d}{dt}$, as the sample positions are advected with the flow, i.e. $\frac{d\mathbf{x}}{dt} = \mathbf{v}$. Thus, it is sufficient to evaluate the right-hand side of Eq. (1) to update the velocity at a Lagrangian sample, e.g. at an SPH particle. In formulations using Eulerian grids, e.g. the stable fluid method [3], the material derivative in Eq. (1) corresponds to

$\frac{D\mathbf{u}}{Dt} = \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u}$. Here, the local derivative $\frac{\partial \mathbf{u}}{\partial t}$ denotes the time rate of change of the velocity at a static sample that does not move with the flow. The advective derivative $(\mathbf{u}\cdot\nabla)\mathbf{u}$ accounts for the advection of the velocity field relative to the sample position. If an Eulerian formulation updates the velocity at a static sample position using Eq. (1), the advective derivative has to be computed and subtracted from the right-hand side of Eq. (1). In the fluid implicit particle method (FLIP) [29], particles are used to compute the advective derivative $(\mathbf{u} \cdot \nabla)\mathbf{u}$.

For multi-fluid simulation, the above equations must be modified corresponding to the target phenomena and scheme. For immiscible fluids, it is possible to apply the single-fluid formulations to each bulk phase and reduce the problem to properly handling boundary conditions at the interfaces. The most popular treatment for simulating a heavy liquid and a light gas is the "free surface" approximation. Here we assume that the gas is light enough that its interactions with the liquid can be approximated with a constant pressure and zero shear stresses at the interface. In this case, the pressure jump at the boundary (optionally with surface tension effects) can be modeled by tracking a single interface, and imposing suitable kinematic boundary conditions [30]. Although this free surface treatment allows for efficient simulations by focusing on the liquid phase, it removes all influences of the gas phase. As such, it does not fit within the scope of this survey on multi-fluid simulations; therefore, pure free-surface simulations are omitted in the following discussion. However, free-surface models can be extended to model certain high-level interactions, e.g. by taking the interface discontinuities into account for surface tensions effects [31], and we will discuss these in more detail below.

For immiscible phases with varying density, ρ can be treated as a spatially and temporally varying function, and incorporated into the discretization of Eqns. (1-2). The coupled simulation of phases [12, 15] is supported at the expense of additional calculations in comparison to the free surface approximation.

For the general case of miscible fluids, a set of extra variables describing local composition of each phase is needed, i.e. the volume fraction or mass fraction field. Denoting the volume fraction of phase k as α_k, the phase-wise Navier-Stokes equation can be expressed as [32]

$$\alpha_k \frac{D\mathbf{u}_k}{Dt} = -\frac{\alpha_k \nabla p_k}{\rho_k} + \alpha_k \mathbf{g} + \frac{\nabla \cdot (\alpha_k \mathbf{T}_k)}{\rho_k} + \mathbf{a}_k^{\text{if}} \tag{3}$$

and the phase-wise mass conservation can be

written as

$$\frac{D\alpha_k\rho_k}{Dt} + \alpha_k\rho_k\nabla\cdot\mathbf{u}_k = 0. \quad (4)$$

Footnote k indicates that the variable is related to the k-th phase and $\mathbf{a}_k^{\text{if}}$ is the interfacial acceleration source that models interaction forces between phases. It is to be noted that in these phase-wise equations, velocity divergences are no longer zero, even if we assume each individual phase is incompressible. As a result, no practical algorithm directly solves the above equations for arbitrary phases.

Several levels of approximation can be applied based on the formulation above. The first one is to adopt a mixture model [18]. Here, an aggregate velocity $\mathbf{u}_m$ is considered:

$$\frac{D\mathbf{u}_m}{Dt} = -\frac{\nabla p_m}{\rho_m} + \mathbf{g} + \frac{\nabla\cdot\mathbf{T}_m}{\rho_m} + \frac{\nabla\cdot\mathbf{T}_{Dm}}{\rho_m}. \quad (5)$$

where a footnote m indicates the variable is an aggregate physical value by fraction-weighted averaging of all the phase-wise values, and it can be treated as the corresponding variables of the local mass center. The last term T_{Dm} is mathematically derived from the phase-wise equations representing convective momentum transfer between phases. According to the mixture model, the volume fractions α_k are updated with

$$\frac{D\alpha_k}{Dt} = -\alpha_k\nabla\cdot\mathbf{u}_m - \nabla\cdot(\alpha_k\mathbf{u}_{mk}). \quad (6)$$

The drift velocity $\mathbf{u}_{mk} = \mathbf{u}_k - \mathbf{u}_m$, defined by velocity difference between phase velocity and the aggregate velocity, can be calculated analytically at each time step under local equilibrium assumption and drag force approximations. Compared to Eqns. (1-2), Eqns. (5-6) each has one more term, which can be calculated from drift velocities on the right hand side, representing the convective momentum transfer and concentration flux due to drift velocity difference. Then instead of all phase velocities $\mathbf{u}_k$, only the aggregate velocity $\mathbf{u}_m$ needs to be solved. The reduced set of equations are easier to solve.

The equations can be further approximated by assuming that every phase always moves together, i.e. $\mathbf{u}_k = \mathbf{u}_m$ for all phases k. Under such assumption, the velocity field is again divergence free and the flow motion can be solved similar to Eqns. (1-2). The fraction change can be independently handled by diffusion models [33, 34] or by an energy-based model [35].

3 Discretization Methods for Multi-fluid Simulation

Fluids are usually simulated on Eulerian grids or as Lagrangian particle sets. Some techniques are directly inherited from their standard single-fluid counterparts to multi-fluid and multi-phase simulation and others are adapted with proper extensions. In this section we provide an general overview of the method-

ologies adopted in graphics for recovering various multi-fluid phenomena, and leave details of these methods to later corresponding sections.

In grid-based simulations [12, 31, 13], as environments containing multiple fluids often need exact tracking of interfaces between different phases, the level set method and its variations are widely used for this purpose, which implicitly defines the interface as zero points of level set functions whose values are positive inside the corresponding fluids and negative otherwise. While dealing with multiple phases, multiple level set functions are needed for each immiscible phase, and function projection must be performed near the joint point of three different phases to avoid voids or overlaps. This projection task can be done by Lagrangian marker particles to detect and reduce the representation errors of the interfaces with the particle level set method [12]. In bubble animation, thin films between bubbles can also be implicitly defined by the interface calculated from proper level set functions in a regional level set method [13, 36].

In many works, especially those involving bubble dynamics, the volume of fluid (VOF) method is applied to handle bubble volume preservation and shape change [37]. VOF uses volume fractions to represent the surface and enforce mass conservation in calculation of the volume fraction changes. It can further be coupled with the level set method as the coupled level set and volume-of-fluid (CLSVOF) method [38] for better stability and detail preservation, during simulation of bubble deformation, collapse and growth.

Particle-based methods, such as the Moving Particle Semi-implicit (MPS) method or Smoothed Particle Hydrodynamics (SPH), are becoming more and more popular for multi-fluid simulation in the past decade [39, 38, 18, 40]. In SPH, physical quantities are carried by fluid parcels - typically referred to as particles - that are advected with the flow [41, 42, 43, 8]. The contribution of each particle to a property are weighted according to their distance from the spatial position of interest using a smoothing kernel function. MPS is similar to SPH, but adds a projection step close to that in grid-based simulators after explicit calculation of particle velocities and positions, in order to ensure the incompressibility of the fluid. Recently, position-based fluid methods are also adopted in multi-fluid simulations that solve incompressibility and fluid motion as an optimization problem over particle positions [44, 45]. Using local calculations instead of solving global projection equations, particle-based methods can more conveniently cope with complex nonlinear fraction transportation formulations in misci-

ble fluid simulations [18, 35, 40] than grid-based methods, which usually use simpler diffusion models [33, 34]. Recently, however, SPH has also been used with global pressure projection [46] and the close relation of SPH to grid-based projection methods, such as FLIP has been discussed [47].

Some hybrid methods also get involved in multi-fluid and multi-phase simulations. Originating from the particle-in-cell method, FLIP uses Lagrangian particles to represent liquid phase and velocity for reduced numerical dissipation [29]. However, incompressibility and motion are still computed on the underlying grid. This method was extended in the form of the multi-fluid implicit particle (Multi-FLIP) method to handle two-way coupling between two immiscible fluids [15]. The material point method (MPM), originally presented as an extension to FLIP and applied to problems such as solid deformation [48], also finds its application in multi-fluid simulations [49, 50]. In MPM, Lagrangian elements referred to as "material points" describe physical bodies. A background grid or mesh is used when gradient terms need to be calculated. This method is promising in handling large deformations in the simulation. Some studies introduce the Lattice Boltzmann method (LBM) to handle multiple phases in the simulation [51, 52, 16, 53]. LBM treats the macroscopic flow as a result of collective behavior of particles performing consecutive propagation and collision processes over a discrete lattice mesh, during which a variety of boundary conditions are naturally accommodated (e.g. pressure drop at interface). In some works, Lagrangian particles are also used for providing in-grid molar concentration estimation, updating the local density[21].

Interfaces between separable fluid phases can be reconstructed in several ways. In level set based studies they are usually directly calculated from the level set functions [12, 31] or by more recent "lattice-cleaving" algorithm [54] for multi-phase level sets. In the works using volume fraction in the calculation, the interface can be reconstructed post-calculation with a marching-cube algorithm [52, 55]. On the other hand, mesh-based algorithms explicitly maintain the interface structure within their frameworks [56, 57].

4 Bulk Flows of Non-Mixing Fluids

Non-mixing fluids, or interfacial flows, consist of those phenomena containing clear and vigorous interfaces between different, usually immiscible phases. Interfacial flows can feature interactive bulk motion, such as in water-oil mixing, or dispersed motion as with bubbles. We restrict our discussion to the for-

mer, where main concerns are to properly track the interfaces between different phases in this section, and to handle discontinuity problems in the interface regions. We discuss the bubble dynamics in §6.

4.1 Grid-based Approaches

For grid-based simulations, the level set method is usually adopted for interface tracking of bulk motions. This seems to be a natural choice, as these techniques originate from single-phase liquid simulations, which are interfacial flows between liquid and air in principle. As in most multi-phase works, motion of all phases are calculated, the impact of quantity jump across interfaces is not omittable, and application of certain single-phase technique is limited in such case. Naive interpolation and semi-Lagrangian advection does not take into account discontinuity at interfaces [12]. Higher-order techniques also face difficulties in early works, as reported in [58], Back and Forth Error Compensation and Correction (BFECC) for interface computation must be turned off near the interface to prevent artificial mixing.

In order to avoid the non-physical smear-out of the discontinuities across the interface, [31] uses a ghost fluid method (GFM) to accommodate jumps in physical values while keeping modifications to the linear system small. Across interfaces where quantity jump exists, ghost values of physical quantities are extrapolated to the other side when calculating differentiations on one side of the interface, which provides convenience. Binary phase interfacial simulations can be successfully performed using this strategy. For more than three phases, in [12], a Poisson equation with discontinuous coefficients is generated in the projection step to incorporate physical jump conditions near the interfaces. A one-sided difference approximation replaces explicit ghost value extrapolation in their derivations, but GFM is still applied to calculate other quantities such as surface tension. Using one-sided difference, at the interfaces where discontinuity occurs, the coefficients are obtained after calculation of the interface pressure and substituted into a flux continuity equation derived from the poisson equation. A level set value projection method is introduced for accurate level set function calculation, both removing overlaps and vacuums between arbitrary number of phases and supporting the aforementioned coefficient calculation. In [59], a soft Heaviside function with a sinusoidal ramp near 0 is used to thicken the interface, effectively providing a smoothed differentiable local function that approximate the numerical jump.

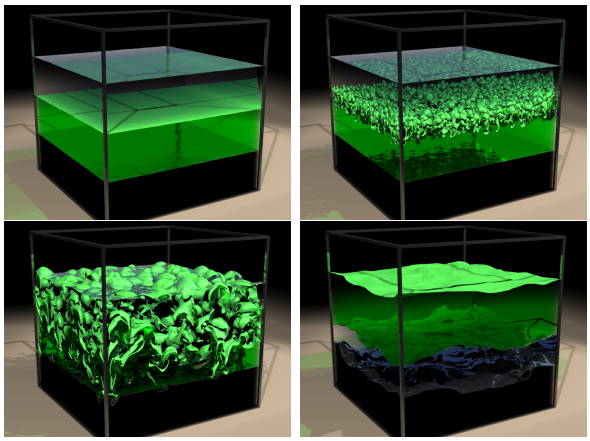

Fig. 2. Grid based interfacial flow simulation result showing Rayleigh-Taylor instability from [12]. Four individual phases are involved in the simulation.

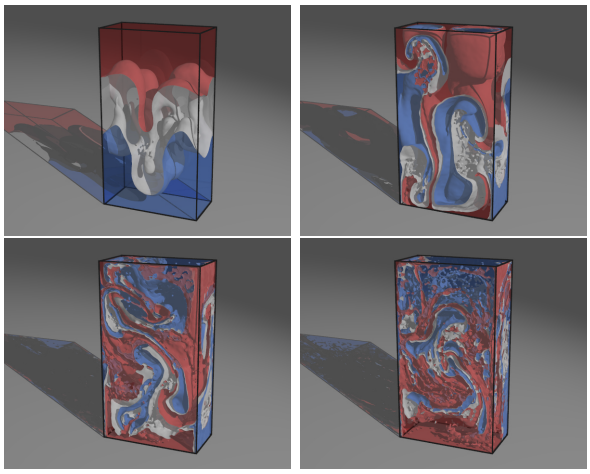

Fig. 3. Particle based interfacial flow simulation result showing Rayleigh-Taylor instability from [14]. Three individual phases are involved in the simulation.

Extending the sign bit in the original level set method to an integer value, the regional level set method is proposed in [13] for better capability to handle fluid components, especially thin films. With their regional level set graph, thin films are captured with graph edges and its density contribution to nearby nodes in the graph is considered in solving the Poisson equation. Thin film rupture condition is also discussed in their implementation. In [60], a real-time simulation technique using sine/cosine transforms is proposed, which is capable of handling multiple fluid simulation under the assumption that all liquids share a common velocity. They also approximate the internal forces between fluids of different density to ensure the incompressibility property still holds in the calculation.

4.2 Particle-based Approaches

In particle-based simulators, a popular approach to handle immiscible fluids is to explicitly calculate the collision responses between particles in different phases. In [61], particle velocities are modified in the SPH simulation based on the momentum conservation law, after collisions between phases are explicitly detected. In their method, a convex hull of fluid-phase particles is needed to detect collision, as well as following particle position adjustment. Another approach to simulate interfacial flows is to simply assign different phase particles with different labels, assigning them with corresponding physical attributes, such as densities, viscosities, etc. The calculation of the

momentum equation remains the same, as long as one can substitute the physical attributes according to the label of the particles. This strategy is adopted in early MPS [39] and SPH [62] works. However, for high density ratios between phases, the MPS method [39] needs an extra iterative process to avoid instability. Similar instability problems in the SPH method [62] are studied in later works, e.g. by Solenthaler [14], where a new density interpolation formulation is introduced to ensure density discontinuity and pressure continuity across the interface during SPH interpolation, allowing density ratio up to 100. Solenthaler's strategy is adopted in later research works using SPH [18, 35] or in position-based methods [44] for multi-fluid simulations. Solenthaler's density contrast has also been combined with a boundary handling that can particularly handle solid contacts with multiple phases [63]. The calculation resolution at the interface can be enhanced using a two-scale particle simulation method in [64], where dynamics near the interface are calculated through a layer of high-resolution particles with proper quantity transition with the other fluid bulks. Using different attributes and governing equations for different phases, solid melting within hot liquid can also be achieved as in [65]. In a power particle approach [66], a pressure projection step is added based on power diagrams, where different densities are initialized to each particle and particle volume is enforced through the calculation for better incompressibility. Their method can apply to interfacial fluid simulations with proper boundary condition calculation at the interfaces. [67] provides a solution for position-based fluid method to simulate immiscible multi-phase flows, extending the constraint calculation for position corrections by taking account of variable kernel size and densities.

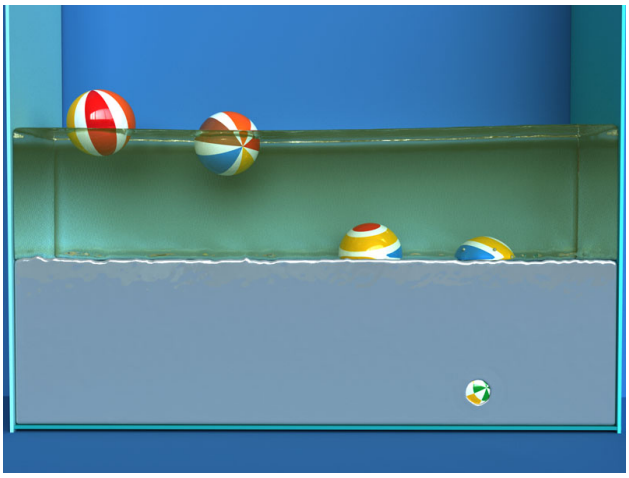

Fig. 4. Two fluid phases with different densities are two-way coupled with rigid spheres of three different densities. In this scenario, density contrast SPH interfaces [14] are combined with multiphase solid boundary handling [63]. Image from [63].

4.3 Hybrid and Other Approaches

Hybrid simulation methods, such as FLIP, are another choice to reproduce bulk interfa-

cial fluid phenomena. While FLIP uses Lagrangian particles to compute the transport of flow quantities, such as the velocity, i.e. the convective acceleration, spatial derivatives for the local acceleration such as the velocity divergence are computed on an Eulerian grid. As an extension of the basic algorithm, Boyd et al. proposed the MultiFLIP approach [15], for which interfaces are reconstructed using Lagragian particles, while a combined divergence-free projection equation handling physical attribute jumps across interfaces is solved on a grid following the ghost fluid method manner. In their approach, particle positions are strategically adjusted near the interface to avoid uncontrolled mixing, which also provides methods for tracking of subgrid bubbles and droplets. As a result, lively liquid-gas two-phase behavior is reproduced. Using a stream function with the FLIP simulation, in [68] two-phase flow simulation can be performed with only the liquid phase calculated and the un-simulated air-phase automatically guarantees divergence-free property through a stream function solver. In their approach air is assumed to be massless in the derivation and consequently the stream function in air region does not influence the calculation. As a result, the method is especially well-suited for liquid-gas two-phase interfacial flows, where gas density is much smaller than the liquid density.

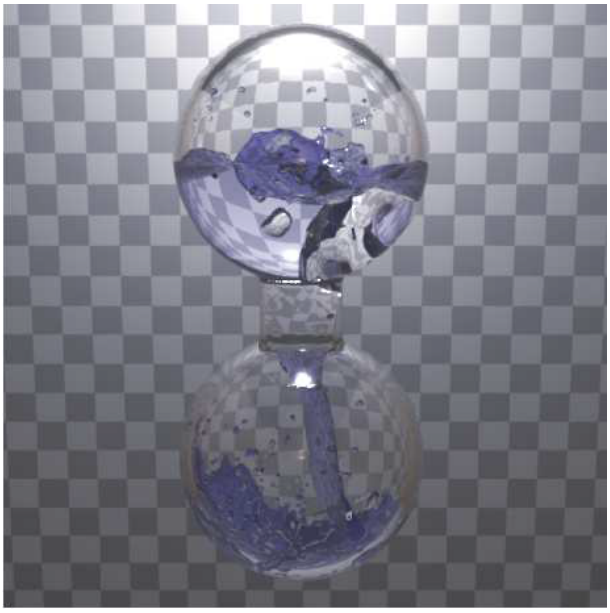

Fig. 5. "Glugging" effect due to air-liquid interaction during water pouring through a spout from [15].

A two-phase LBM model is proposed by [53] for interfacial flow simulation. Compared to previous LBM methods, sharp interfaces are achieved in turbulent interface dynamics as a result of applying a mean-field model. Overshooting of physical quantities is suppressed by modifying viscosity and pressure near the interface. They also provide a detail preserving method in simulations of two-phase flow under high Reynold number.

In [56], interfaces between different phases are directly represented by unstructured tetrahedral meshes, whose dynamics are computed through a series of deformation algorithm-

s. Solving of fluid simulation is treated as a quadratic optimization problem in their work, and while dealing with multiple phases, exchanging of volume between different phases is avoided through adjustment in the pressure calculation. Such FEM scheme later also finds application in single-fluid simulation [69]. [57] also represents multi-phase flow by tracking the surface with mesh-based manner. Various mesh operations are considered in order to preserve watertight, intersection-free meshes with potentially complex topologies. In [70], the multi-phase interfaces are explicitly tracked with the help of an additional indicator function. They compute the multi-phase interface in a two-step manner. Narrow-band surfaces near the actual interface is first constructed, then the exact interface is marked from the Voronoi interface of these surfaces. A "lattice cleaving" algorithm is proposed in [54] for generating tetrahedral meshes of multiple materials. Based on volumetric indicator functions, this method considers complex mesh stencils in local tetrahedral cell to correctly recover interfaces between multiple phases, and well-shaped conforming mesh discretization is generated for calculation of fluid physics.

5 Volume-Fraction-based Fluid Mixing

Continuously changing mixtures of fluids, whether in a temporarily mixed manner as with a suspension, or in a permanent miscible manner as with solutions, cannot be represented by a single discrete interface. When different phases mix together, the fluid states are usually described with the assistance of volume fractions. Various physical quantities, such as the aggregate density, are computed from single-phase values using fraction-averaged calculation. One key issue in computing miscible fluid dynamics is to evaluate the concentration changes of different phases during fluid motion. For this purpose, using a diffusion model can reproduce nice, one-directional fluid mixing effects, while more complex models, such as a mixture model or energy-based model, can further recover more real-world behaviors, such as phase-separating of suspensions.

5.1 Diffusion Models for One-Way Fluid Mixing

Using the volume-fraction representation, it is relatively easy to recover one-direction fluid mixing between different phases. In graphics, a common strategy first described in [62] and adopted by many works is to add a diffusion equation to the volume fractions, balancing out the concentration difference during the simu-

lation. In the grid-based methods [33] using LSM or [55] using VOF, for multiple miscible and immiscible mixing, phases are divided into different "miscible groups" where only diffusion between phases in the same group is allowed.

Similar strategy is adopted in particle-based methods [62] and [71]. In [72], fluid particles in simulation are defined as surface particles and bulk particles for detergent mechanisms, and concentration change is categorized into adsorption, diffusion, cleansing and coating effects, each related to one specific fluid transport phenomena.

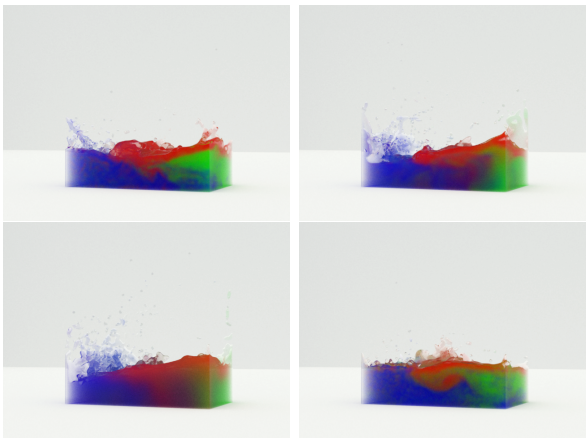

Fig. 6. Different three-phase mixing effect from [18] achieved by varying settings in a mixture model. Top left: all phases immiscible; top right: all phases miscible; bottom left: all phases miscible with additional diffusion effect; bottom right: red and green phases are miscible to each other, but immiscible to blue phase.

In [73], a hybrid approach for multiple fluid handling is proposed. In their method, the main bodies of each phase are treated as immiscible fluids and solved by the grid-based method from [12]. To handle chemical reaction and diffusion effect near the interfaces, Lagrangian concentration particles are used to track the local concentration of each phase. As an early solution, their results provides certain multi-fluid effects including reaction, diffusion and viscous fingering, but has layered visual appearance in the results.

Ink motion is an interesting phenomenon where it appears like diffusion at macroscopic scales while the ink particles are actually in an immiscible dispersed phase microscopically. The LBM method is adopted in [74], where pigment concentration is calculated on surface layers to model multi-color ink dispersion in absorbent papers. In [75, 76], the volume-fraction based ink diffusion effect is compared to a hybrid grid-particle method, where highly detailed ink effects are controlled and visualized by particles interacting with low-resolution grid fluids and solids. They show that simple diffusion mixing is insufficient in preserving details for ink dispersion in liquid.

5.2 Unified Models for Immiscible and Miscible Dynamics

Temporary mixing where phases unmix from each other in the end are harder to capture

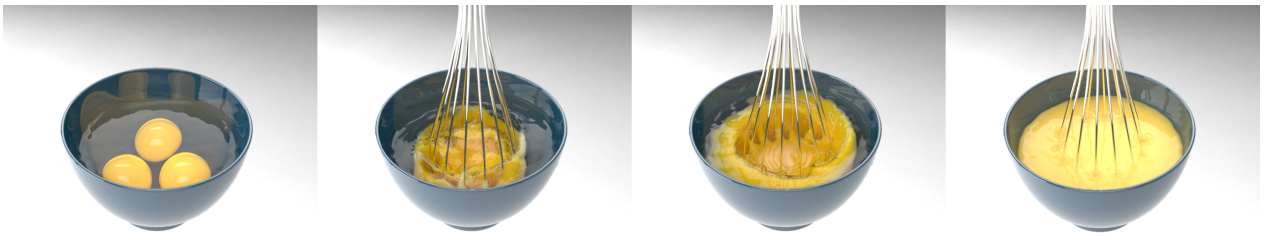

Fig. 7. Simulation of an egg mixture from [35] using an energy based Navier-Stokes Cahn-Hilliard model.

and may need more complex models, such as the mixture model [18] or Cahn-Hilliard model [35], which are all particle-based method in the graphics literature. These models usually cope with immiscible/miscible and mixing/unmixing phenomena in a unified manner and reproduce the phase-separating effect from immiscible settings of the phases. In [18], based on Weakly Compressible SPH [42], a drift velocity defined as the relative difference between phase velocity and the aggregate velocity is analytically calculated, driving immiscible and miscible phases to mix or unmix with each other dynamically in a unified manner. It is extended in [40] to provide unified simulation containing both liquid and solid phases. In [35], the mixing and unmixing are modeled using an energy-driven concentration change process with lower computational cost, also supporting the position-based fluid method [44]. This model is later enhanced to handle solid phase and its dissolving in liquid in a unified manner by introducing a state energy function taking account for both phase field and concentration fields [77].

5.3 Other Approaches for Fluid Mixing Effects

Before the volume fraction representation is widely adopted for miscible fluids, the following works study the miscible fluids phenomena using methods such as LBM and gradient flow. In [78], a two-fluid LBM method is introduced to handle double-phase mixing of miscible fluids. Later in [16], the method is stabilized with subgrid model and is enhanced for free-surface capturing. The dynamics of the fluid is divided into self-collision and cross-collision in these works, however the complexity of related terms make extension to more than three phases non-trivial. This issue is solved in the later work [52] using the volume fraction description, where LBM is combined with the Cahn-Hilliard model. Interfaces between immiscible phases are also recovered in a unified manner in this work and reconstructed with the marching cube algorithm. A method based on gradient flow is proposed in [79], which is also able to cope with

mixing of multiple phases.

There are also studies dedicated to two-phase simulations for specific simulation purposes. In [17], a two-phase flow model fully considering the phase-wise dynamics is proposed to simulate water spray. The phase-wise equations are solved on an Eulerian grid implicitly. By introducing an enhanced two-phase diffusive flow model into grid-based simulation [80], controlling of thickness of diffuse interfaces can be achieved, providing visual effects from mixed-phase state to separated-phase state in two-phase simulations. Anisotropic diffusion effects can also be reproduced by artistic control. In [23], a two-phase Shallow Water Equation model is provided to handle dissolving and re-sticking of the solid phase by thin liquid on curved mesh surfaces.

6 Bubbles

Bubbles are commonly observed in real-world fluids. The appearance of bubbles is very diverse. Large bubbles change their shape during rising in the liquid, vibrating, or even splitting, while smaller bubbles mostly remain in spherical shape. Bubbles usually emerge from trapped air during liquid motion, boiling, or escape of dissolved gas. In these situations, bubbles can significantly enhance visual appearance in 3D graphics applications. As a result, the simulation of bubbles remains an active research topic in visual simulations of fluids. Efficient modeling of sub-grid bubble shapes and dynamics are of particular interest on this topic. In the graphics literature, Eulerian methods tend to be adopted for large bubbles, while dynamics of smaller bubbles is often handled in a Lagrangian formulation.

6.1 Large Bubbles

Large bubbles can be simulated in a similar way to bulk interfacial flow, as long as bubble sizes are big enough to be described within grid-size limitation (e.g. [31]). Under VOF framework, [37] uses a front-tracking method to reduce numerical diffusion in bubble deformation, and reproduce the motions of a small number of bubbles rising and merging with free surface or each other. In [36], the regional level set method is used to simulate bubble stacks and clusters. A regional distance function is proposed to help develop an interface tracking technique between bubbles. Surface tension is also exploited in the work for calculating correct bubble shape in the stack. Later, [81] introduced a volume control strategy in the regional level set method to carefully maintain bubble volume during the simulation.

6.2 Sub-grid Bubbles

The above methods mostly simulate bubbles using Eulerian grids, while small bubbles of sub-grid size are hard to capture. To solve this problem, [82] proposed to identify sub-grid droplets and bubbles from local near-zero level set function values. By incorporating sub-grid refinement of the level set with the VOF method, a geometry-aware volume-of-fluid method is proposed in [83], which successfully recovers deforming bubbles with both large and small sizes.

Particles have been used for a long time for bubbles in graphics. [84] creates bubbles from air marker particles escaped into liquid, with overlaps handled through a rendering step; [85] in turn uses physical quantities such as the Weber number and uses marker particles to identify and simulate possible bubbles and droplets. In those works and related research, the dynamics of sub-grid bubbles usually contain buoyancy and quadratic drag force due to interaction between liquid and bubble (e.g. [85, 86]), and certain cohesion forces that act between bubbles modeling the self attraction of them within the liquid (e.g. [84, 19]). In these models the simulated bubble number can be large, reproducing satisfactory visual effects, but the dynamics and interactions are still relatively monotonous with a closer look. To recover the commonly observed unstable phenomena such as zig-zag or spiral path and bubble splitting, [87] seeds bubble particle with random movements within the air bubbles and track them in an adaptive grid.

6.3 Foamy and Dispersed Bubble Flow

Large amounts of small bubbles can have a foamy appearance, or be dispersed dynamically in the liquid, e.g. [88]. These kinds of bubbles are usually treated in a Lagrangian way. In the former case, self-clustering and cohesion with liquid and solid are investigated in [19] with various cohesion forces and a volume-preserving simulation framework. The stack interface structure is modeled with help of a Voronoi diagram generated from bubble particles. In the latter case, local volume fraction of the bubbles can be calculated to affect the aggregate density in the liquid projection solver [89]. An SPH-based simulator can also be used that directly simulates discrete bubble particles with additional buoyancy and drag forces between fluid particles. [86] uses spherical single particles obtaining volume from dissolved gas in liquid to represent bubbles; a similar strategy that models gas particle generation from gas concentration increase at nucleation sites (randomly or artificially selected) surrounded by gas-carrying water particles is proposed in

[90], with additional consideration of drag and cohesion forces between bubbles, liquid and solids. In [91], a bubble can consist of multiple SPH particles detected from trapped air, in this way supporting more strongly distorted shapes. This method also proposes an updated drag-force formula that is used in the two-way coupling of the fluid and air phases. In order to avoid instabilities at the fluid-air interface, both phases are simulated separately and two-way coupled. The air phase is simulated on top of the fluid phase, ending up with air and fluid particles at the same positions. This allows for an independent and stable simulation of both phases, while interfacial forces are computed at the surface of air bubbles. These forces are used for the two-way coupling of both phases.

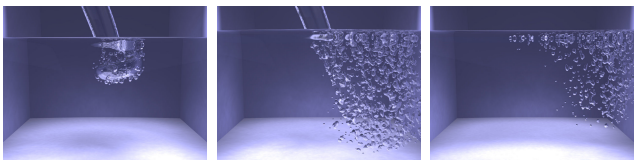

Fig. 8. Air and fluid phase are simulated independently. The air particles are generated and advected on top of fluid particles. Both phases are two-way coupled. Image from [91].

As MPM supports a wide range of material behavior, it was used to model the behavior of dense liquids foams in a continuous setting. In [92] the authors propose an MPM based approach that models finely dispersed bubbles in a fluid with a shear-dependent material model. The MPM approach in [93] is based on the viscoelastic-flow Oldroyd-B model instead, but similarly proposes a formulation that can handle foam materials as a continuous viscoelastic effect. Both approaches do not model bubbles explicitly, but compute their averaged contributions to the bulk flow within the MPM framework.

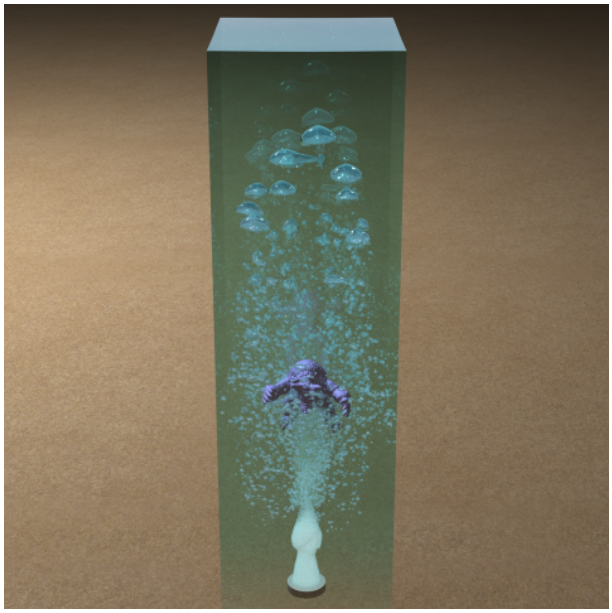

Fig. 9. Bubble dynamics from [20], flexible bubble volume change between large and small bubbles and two-way coupling to the surrounding flow are achieved by their hybrid method.

6.4 Hybrid Methods for All-size Bubble Modeling

Hybrid simulation methods bring more balance in simultaneously simulating large and small bubbles. In these methods large bub-

bles are simulated using a grid-based integrator, while small bubble motions are captured in a Lagragian manner recovering better small-bubble interaction in the densely dispersed case. [94] gives a practical two-way coupling strategy between the two kinds of simulators, and [95] seeds control particles experiencing non-uniform drag forces within large-bubble regions to better recover realistic ellipsoidal bubble shapes. Later [20] uses a semi-implicit compressible flow model for sub-grid bubbles and couple them with incompressible level set solver in a divergence contribution term, allowing both bubble volume change and monolithical two-way coupling to the surrounding flow. A series of pre-computed bubble shape templates are used for the dense bubbles saving the effort to compute individual small-bubble shapes. These pre-computed templates are purely used for rendering and do not get involved in the simulation. Coupling with solid and bubble seeding depending on local vorticity are also analyzed.

6.5 Sprays and Mists

Generation of water sprays and mists are usually handled in a similar manner. In [97], they are represented by introducing Lagrangian particles to a VOF simulator. In [98] Lagrangian particles are again used for foam, spray and mists, with foam particles sticking to the liquid surface, while the other two kinds of particles are moving ballistically. On the other hand, sprays can be simulated using an augmented coarser grid as in [99, 100], or using a narrow-band of air particles near liquid surface as in [101], however such strategies are not able to reproduce general motion of spray without using a full multiple fluid model [17].

6.6 Post-processing techniques and other bubble effects

Some research adds bubble effects as a post-processing step to a primary simulation. In such works usually the added bubbles have dynamics influenced by the liquid but not vice versa. [102] generates diffuse particles from particle-based simulations. Classified into spray, foam and air bubbles, diffuse particle generation are controlled by designed potentials calculated from the liquid motion state. Motions of the diffuse particles are determined using the fluid velocity and external forces. Such a technique can add rich spray appearance to a pre-obtained liquid simulation result. This work is extended by [103] to bring detail enhancement to FLIP fluid simulations. Later [104] provides a bubble recovering methods that can extract dense bubble motion from multiple-fluid simulations. Both Eulerian

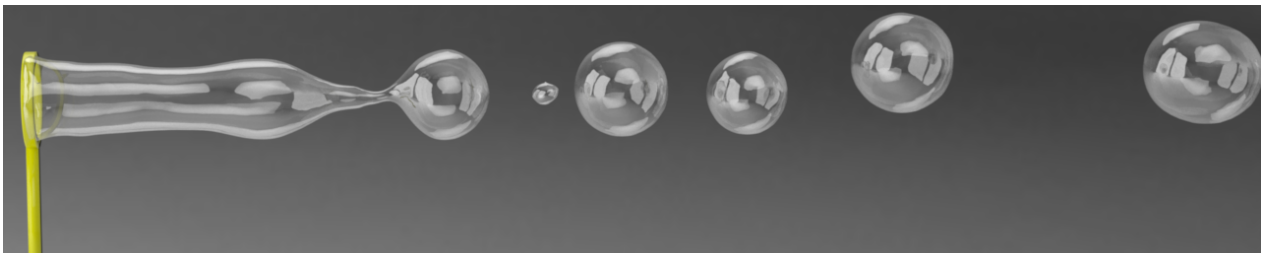

Fig. 10. Particle based air bubble simulation from [96]. Bubble shape is maintained through two-phase pair-wise surface tension force.

method and Lagrangian method are supported in their approach.

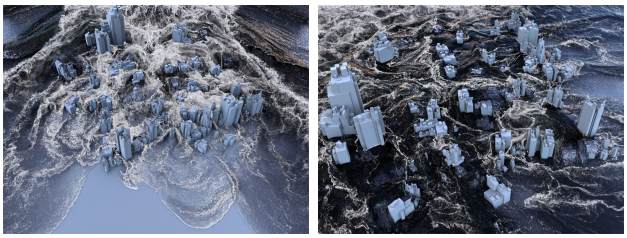

Fig. 11. SPH water with particle-based foam. The unified foam approach from [102] is rendered with a screen space technique from [105], which is also the origin of these images.

Bubbles are also studied in certain works with specialized algorithm frameworks. [106] considered bubble dynamics in a Shallow Water Equation (SWE) framework, where bubbles within the liquid are simulated with particles and analytical vortices, while foam particles are simulated with SPH. Gas bubbles in air such as big soap bubbles are visually interesting, and they can be reproduced by codimensional surface tension flows [107], vortex sheets [108], or hyperbolic mean curvature flow [109]. In particle-based models [110, 96], gas bubble dynamics in air are modeled with two-phase interaction calculated on the outermost layer, with a pairwise force technique for surface tension. This strategy of calculating molecular cohesion can also be found in single-fluid work [111]. Bubbles are also an important component of boiling simulations [62, 26, 38]. The bubbles are handled similarly with Eulerian big bubble techniques in 6.1, and these works will be discussed in more detail in §7.

7 Dissolving, Reactive, and Phase-Transitional Multi-Fluids

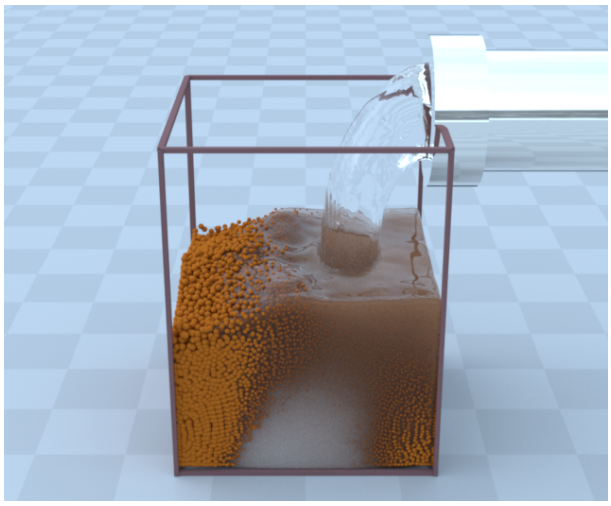

Fig. 12. Simulation of coffee and candy dissolving in water from [40]. Solid and liquid phases are treated in a unified framework.

The problem of dissolving, reactive and phase transitional fluids are complex in that they can not only be multi-fluid but also involve state change in one or more phases. To describe such phenomena, equations calculating temperature or state-energy changes often appear in the models.

7.1 Suspensions and Dissolving Multiple Fluids

Suspensions and dissolved solid in liquids are often studied in a multi-phase manner. Described by the concentration of sediment mass, solid erosion is simulated using a sediment transportation equation in [112]. Cohesive and cohesionless materials are treated differently in this approach. Erosion is combined with rigid-body motion calculation in [22]. An additional density field exists here for rigid body dynamics and density-dependent surface reactions. Solid volume fractions are calculated to reproduce sedimentation effects in their work. [113] treats the water-sand mixing process as coupling of porous material with fluids, and uses porous flow simulation to reproduce muddy flows. [114] combines Lagrangian suspension particles and level set method for muddy liquid simulations. The suspension particles are sampled near the liquid-solid interfaces and their motions are considered to follow similar rules with bubble dynamics in previous works. [40] adopts the mixture model on solid-liquid mixtures by introducing a mixture deviatoric stress tensor for the solid phase. In this approach, dissolving of solid into liquid is captured by volume-fraction change driven by a Noyes-Whitney equation. Porous sands and their suspension in water are also studied using MPM in [50]. It uses separate grids for liquid and solid motions and an effective model is derived to handle momentum exchange in overlapping regions of the two grids. Sand cohesion is linked to the water saturation at the corresponding region. [23] considers solid dissolving by thin liquids moving on arbitrary triangle

meshes and by providing a two-phase shallow water equation method.

7.2 Reactive Fluids

Reactive fluid is a typical multi-phase phenomenon. In [115], reaction between the paints in marbling is recovered based on a Fitzhugh-Nagumo model for spiky dynamics generation. The result depicts visual appearance of 2D immiscible flow. [34] adopts molar concentration models to recover chemical reaction in multi-phase flows on a grid-based solver. Density in the projection is averaged from the molar concentrations, which in turn is calculated independently with fixed reaction rate. [21] adds Lagrangian particles to support better prediction of molar concentration and increase computational efficiency. In [22], solid level set is advected proportional to the reaction rate to reflect body deformation during acid corrosion. [18] provides a simple chemical reaction handling strategy as an in-particle re-balance step in SPH methods. The concentration change can be measured locally with volume fraction calculation within the mixture model framework.

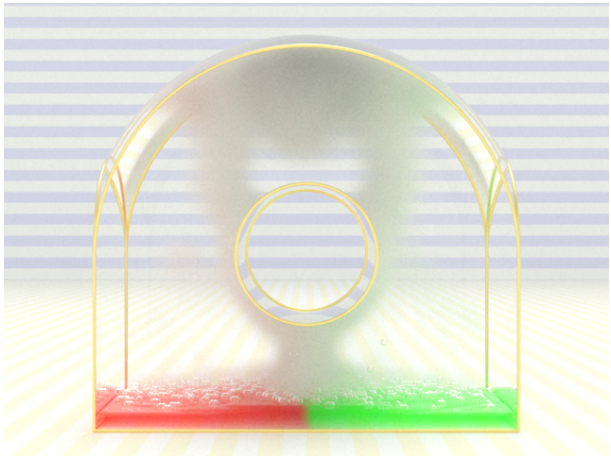

Fig. 13. Two reactive fluids meet and produce gas from [18]. Four phases are involved in this simulation.

7.3 Phase transition in Multi-fluid Environment

Phase transition is extensively studied in computer graphics. It often involves multi-fluid simulations or uses descriptions inherited from multi-fluid models. One such example is the burning effect, which falls in the solid-gas transition category. A two-phase model is adopted in [6] for fire simulation, where quantities of gaseous fuel and product are separately stored before being integrated into the Poisson equation in the projection step. In [116] suspended fuel particles are explicitly tracked in the gas simulation; when ignited these particles begin combustion and produce explosive results. In [24] a particle level set method is adopted to track the solid-liquid-air transition at the interface during burning and melting.

In this method, particles are labeled to different phases and are converted to other phases during phase transition.

Solid-liquid transition includes melting and freezing. Among this topic, several works in graphics are developed within a multi-fluid environment. An LBM-based method is proposed to handle melting and flowing in [51]. The fluid is separated into solid, liquid and gas phases. Within such multi-phase environments, the cell modes transition between different solids and liquids is controlled by temperature using a threshold. Modified collision rules are used for the solid-liquid and liquid-gas interfaces to prevent particle penetration between different phases during fluid motion. In a level set method [117], movement of dissolved air is considered in water freezing. The radii of air bubbles are calculated based on the pressure of water and generated during the freezing process. The local amount of uniformly distributed air bubbles reflects the dissolved air concentration before freezing, generating ice appearance containing large amount of bubbles. In [25], the same problem is investigated in a particle-based formulation using the position-based fluid method. Compared to the previous method, air bubble diffusion is also calculated, resulting in a more realistic visual appearance of bubbly ice. The diffusion can also be controlled by a threshold, leading to various freezing speeds in the simulation.

Liquid boiling is common in daily life but is hard to realistically reproduce in computer graphics, since it involves both physical phase transition and gas-liquid multi-phase interaction. The SPH method [62] begins with transforming liquid particle to gas particle at a certain probability when temperature reaches the boiling point to mimic slow boiling near a heating source. [38] uses CLSVOF to simulate bubble dynamics during boiling. A physically-based mass transfer equation is solved at the gas-liquid interface. Detailed entangled recipes are given in solving heat flux and momentum equations after seeding bubble regions on a solid surface. [26] uses a Yanagita model that apply a handful of simple operators to a single heat field and determine whether a cell should convert from water to steam or vice versa. The model is carefully extended to model surface tension and to be coupled with the particle level set method. An SPH method is adopted again in [27], where a thermally driven bubble expansion model is given to model bubble growth in superheated water after nucleation.

In addition to boiling, vaporization and the formation of clouds are also related to liquid-gas phase transitions. An early book on this topic is [118] where systematic recipes

on cloud dynamics including fluid flow, water continuity and thermodynamics are given, along with description on cloud rendering techniques. In [119], vapor density and saturation steam density are both taken into account for cumuliform cloud growth. The transition rate is linked to the heating and cooling by an energy function. [120] follows the same theoretical formulation and adds a controlling mechanism in the simulation. A geometric potential field defined by the target shape attracts the vapor, and with the help of a latent heat controller and as well as a water vapor supplier, it manages to generate clouds with given target shapes. [121] proposes to add water vapor proportional to the temperature, i.e. changing the saturated water vapor density according to temperature change during the vertical cloud development. As a result, it achieves faster convergence in the resulting algorithm. A two-fluid model is applied in the modeling of volcanic clouds in [122], where the evolution of magma and entrained air are separately computed using the mass conservation equations. Then other physical quantities are averaged according to local magma and entrained air amount and applied in calculation of an incompressible Navier-Stokes equation. In [123], a particle splitting and merging strategy is proposed to enable stable liquid-gas phase transition in SPH simulations. A pattern-based scheme designed to minimize the change of local mass distribution is derived, and it can successfully simulate vaporization in large, complex scenes.

8 Single-fluid Phase Transition

As discussed in §1, the multi-phase phenomena of the real world do not necessarily fall in the multi-fluid category, with most of the exceptions here belonging to single-fluid phase transitions. A number of computer graphics works derive physical models for phenomena from this area, e.g., melting, freezing and burning effects. They are also worth mentioning in this section in comparison to those that use multi-fluid models in §7.

8.1 Melting

The single-fluid melting process can be modeled by changing material attributes according to the temperature at the current state, whether directly changing state at a certain temperature or by interpolating between set values at high and low temperature thresholds. In [5], the solid and liquid phases are separated using different viscosity values. A high-viscosity matrix solver is derived for solving the Navier-Stokes equation with variably high viscosity and recover melting and hardening effects. The particle-based method [124] model-

s melting effects as an extreme case of plastic deformation, where the deformation displacement is always absorbed and considered zero. Whether the material is in a melting state is left to be defined by the user. [10] further varies a set of different attributes including stiffness, compressibility, plasticity, viscosity and surface cohesion according to temperature to discriminate solid and liquid phases. Later, [125] incorporates temperature-based viscosity change into an SPH viscoelastic model to recover melting and flowing phenomena. [124, 10] also uses a surface element technique for solid surface detail preservation and flexible topology changes. In [126], a thermal model including radiation is adopted for ice melting. This work also enhances the VOF algorithm for better surface tracking of solid phase. [49] extends MPM to capture phase transitional effects. A latent-heat buffer is associated with the particles to model constant temperature during melting and solidification at the melting point, i.e. the particles only change to solid/liquid state, when the buffer is empty or full at the melting point. [127] solves the dynamics of the fluid phase with an FLIP solver, and the solid phase with position based dynamics, with phase change determined by a temperature threshold. A mesh-based method is proposed in [69] to simulate liquid and solid phases in a unified framework, with efficient dynamic local re-meshing schemes for topology changing during melting. In this work, the driving mechanism of melting is modeled by heat-contributed strain in the solid, whose increase at high temperature results in high strain rate and makes a material with high thermal expansion melt.

8.2 Crystal Formation

Crystal formation in solidification is an interesting phenomenon, where structured solid phase appears during the process. [128] introduces an anisotropic model to reproduce dendrite growth in a grid solver. It adopts the phase-field model and calculates the phase change between solid and liquid phases through a continuous phase field. Anisotropic functions for phase-front energy is designed to control the symmetric growth direction, in order to recover real-world ice crystal shapes. Bump-map generation technique on the crystal surface is also studied to provide better appearance in the final rendering. [129] enhances the simulation of dendrite growth by combining the phase-field model to a diffusion limited aggregation model. As a result, symmetry-breaking patterns are introduced and more realistic dendritic growth are simulated. The growth of 3D icicle is treated as a thin-film Stefan problem in [130], where

a level set solver is adopted. The rendering is enhanced with growth time and the liquid velocity on the icicle to provide realistic results.

8.3 Burning

For the case of burning, although it is potentially linked to multi-fluid reactive simulation or phase transitions, aside from those mentioned in §7.3, it is handled in a simple way by incorporating brightness or color based on calculated temperature with a pure single-fluid Navier-Stokes solver in many graphics works [131, 132, 133, 6, 134]. Explosive fluid behavior is controlled to follow designed path or shapes in [135, 136], and generation and dynamics of fire-flakes are studied in [137] to enhance the flame shapes for visual effects.

9 Capture-based Methods

In multiple fluids and their mixture, complex patterns and colors often appear in their flows. Fluid texturing techniques that introduce complex surface details via temporally-varying texturing on dynamically evolving fluid surfaces can be used to render such effects [138, 139]. In addition, capture-based methods can also be used to achieve similar visual effects. They provide highly, or even more, realistic results from a different perspective other than pure simulation.

In [140, 141], the fluid simulation problem is combined with tomographic scanning. The fluid velocity field is reconstructed by the tracked flow, providing realistic result of miscible smoke flows. Their methods can reproduce motions and eddies based on real flows, but requires series of volumetric data-sets for reconstruction. Approximate fire and smoke motions are recovered in [142] from multi-view videos. They use an appearance transfer method for the fluid-volume modeling, and can add plausible details to the fluid animations as a consequence.

To capture the behavior of liquids, Wang et al. [143] scan diffuse surfaces of liquids and reconstruct their motion over time with a simulation. This yields realistic liquid motions with full air phase interactions at the expense of a complex capturing setup. They also rely on opaque liquid surface, which means that the interior bubbles cannot be geometrically recovered, and the appearance can be similar to that of a single-phase fluid.

10 Summary and Future Directions

In this survey, graphics studies on the multiple fluid simulation are discussed. The governing theory, discretization method are also presented. A number of techniques on the multiple-fluid phenomena, including im-

miscible and miscible flows, bubbles, physical or chemical phase changes and capture-based methods are covered in this survey.

Multiple fluid phenomena are challenging in the complexity of their description and rewarding in their ability to produce highly impressive visual effects. Conventional problems in single-fluid simulations, such as solid-boundary handling, detail enhancement, etc. remain important in multi-fluid studies. However, multi-fluid simulations introduce new challenges and topics that need considerable attention, such as interface-tracking, concentration calculation, attribute-change handling in various physical and chemical procedures, just to mention a few.

In the current multi-fluid literature, grid-based methods (e.g. [12]) are still taking the lead in better incompressibility and digital stability, while particle based methods (e.g. [18]) are easier to handle complex governing equations and are more friendly to GPU parallelization. Hybrid approaches are often adopted thanks to their accuracy in interface tracking [15] and ability to uniformly simulate multi-fluid dynamics within large scale range [20].

Among the various methods derived to recover real-world multi-fluid phenomena, several are of great interest to graphics simulation. Variants of level set methods play an important role at locating the interface between immiscible components ([12, 13, 20]). The concept of volume fraction are becoming more and more popular for all kinds of multiple fluid simulation. It can be used in pure liquid simulations, such as in VOF ([38]) for reconstructing surfaces and preserving volume, in miscible flow models for computing component concentration changes([18]). It can also describe dispersed or dissolving phenomena from a macroscopic statistic perspective, and has been used in related works ([40, 50]). Many reactive process also relies on fraction-based calculation ([34, 18]). An energy function has been first adopted to describe phase transitional or reactive phenomena , however recent studies show energy-based models are promising in efficient calculation of general complex multi-fluid simulations ([35]). On the other hand, capture-based methods provide another dimension of recovering highly realistic multi-fluid effect other than pure simulation [141], which can easily outperform the latter in a sense of effective resolution, but at the expense of complex capturing setups.

There are several unresolved issues in the current literature on multi-fluid simulation. The first one is simulation stability under high density ratios. A high-density ratio not only leads to smaller time steps, but it also causes

detail loss in miscible simulations: a cell or particle with slightly smaller aggregate density can be mostly occupied by gaseous phase, and the details of the shape of liquid phase (which only occupies a small fraction of the local volume) is lost. This problem has been better handled for interfacial flows (e.g. [14]), but is more difficult for miscible flows, especially in cases of reactive or phase-transitional fluids, where dramatic change of densities between the original phase and the product results in volume explosion.

The second issue is the mass conservation problem during multi-fluid simulation. In some two-phase bubble and interfacial flow works, VOF can be adopted to handle the volume/mass conservation of a certain phase. However, it is not yet resolved in a general sense, e.g., simultaneously capturing detailed interfaces between multiple immiscible phases with mass preservation, or ensuring strict mass conservation during concentration calculation in miscible simulations.

A related challenging topic to visual simulations of multi-fluid simulations lies in the photo-realistic rendering of the simulation data. As multiple fluid phenomena can feature complex tangled surface structures and rich color variations, rendering them requires multi-refraction between the surfaces, and often involves volumetric effects. The latter can be very time-consuming, as compared to single-fluid results that primarily consist of surfaces. Thus, there are a variety of new challenges on the rendering of multiple fluids, as well as the simulation-related problems that present interesting avenues for future research.

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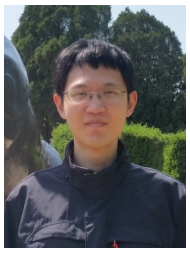

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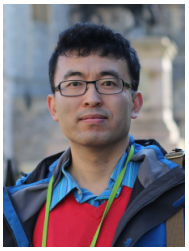

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